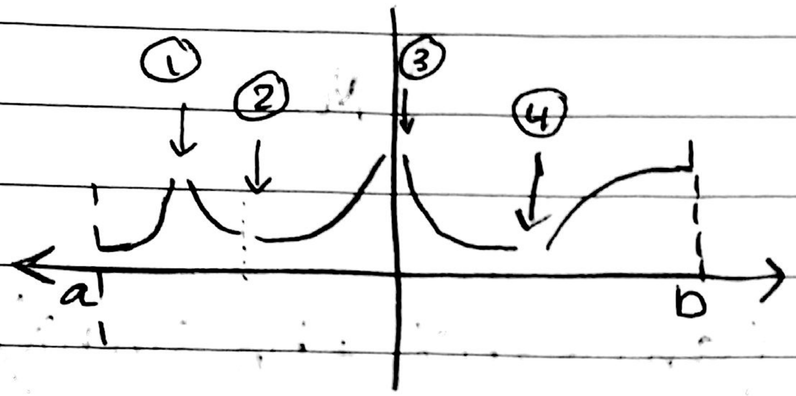
Lec1

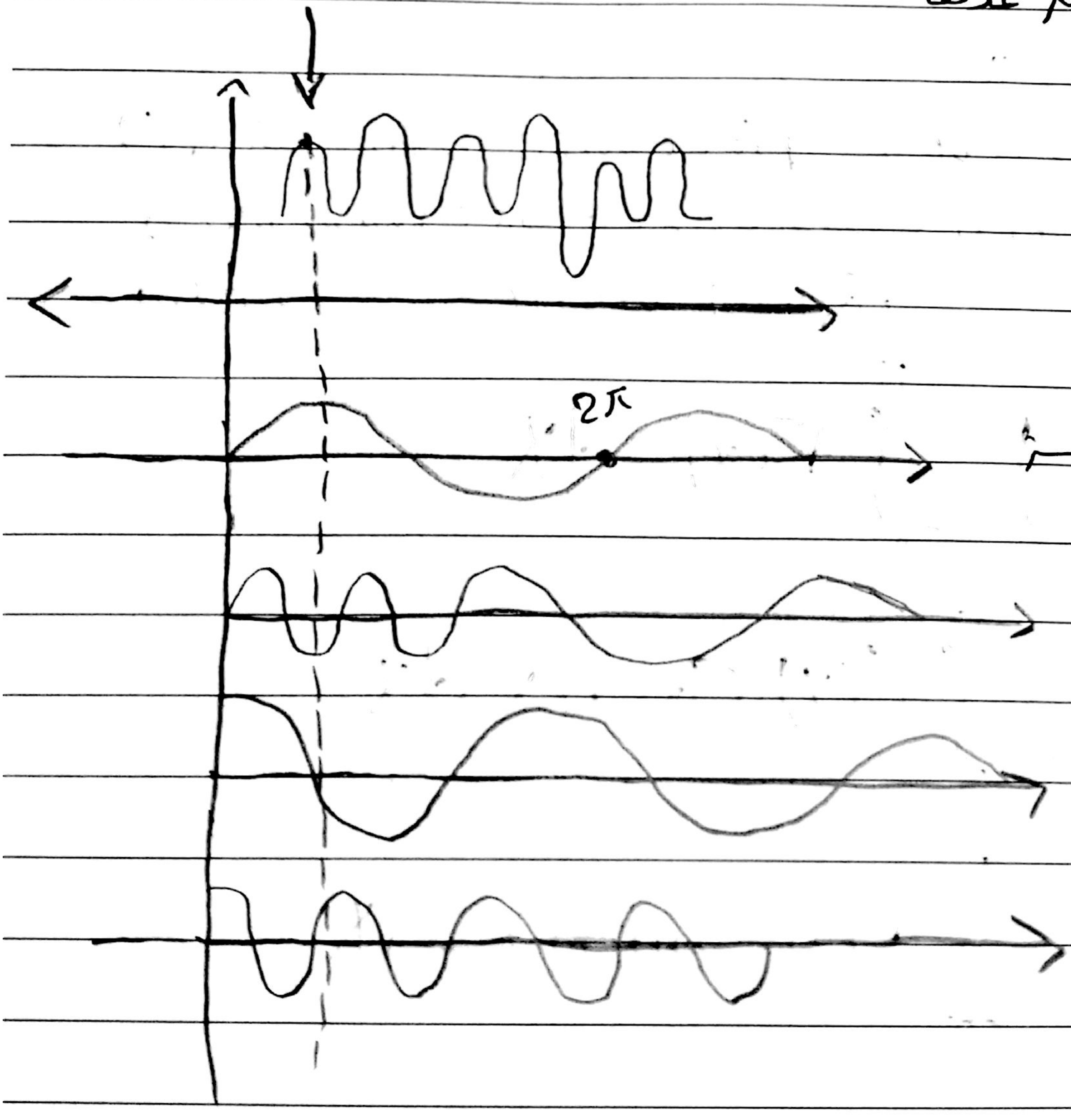
Mathematics

Fourier series

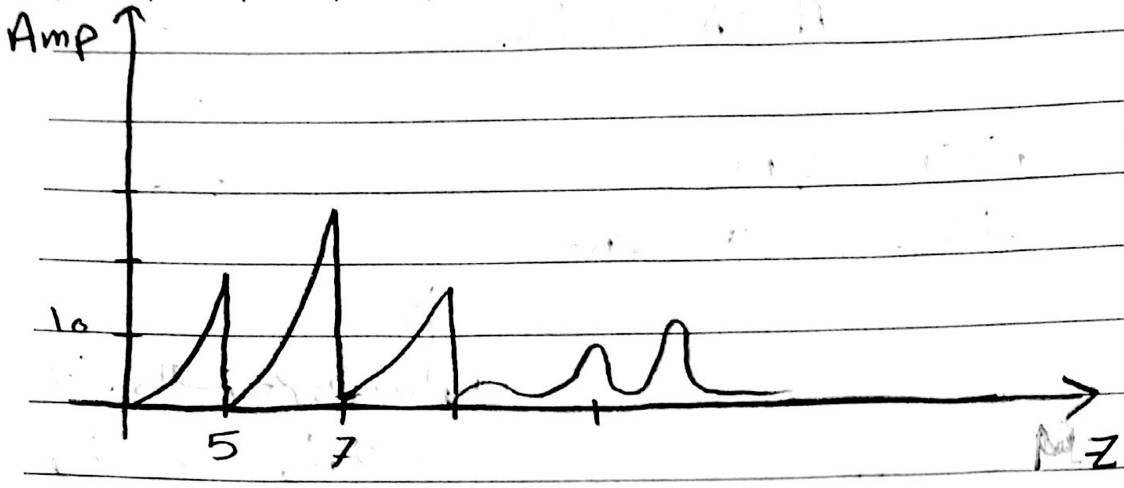
* piece wise Cont Fn.



لأي دالة معروف بدايتها
ونهايتها تؤول لأية
محددة عند البداية
والنهاية
ونقاط عدم الاتصال
فيها يمكن عدها



Fourier
بيفكها لمجموعة
من
Sinx
أو
CosX



spectrum

$$* \quad F(x) = \frac{\overset{\text{ثابت}}{a_0}}{2} + \sum_{n=1}^{\infty} \left(\underbrace{a_n}_{\text{ثابت}} \cos nx + \underbrace{b_n}_{\text{ثابت}} \sin nx \right)$$

at $P = 2\pi$ $\qquad a_1, a_2 \dots \qquad b_1, b_2, \dots$

لأى دالة تكون piecewise وتكون دورية

[تعيد نفسها كل فترة زمنية معينة $F(x+P) = F(x)$]

تمثلها ب Fourier series

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x)\, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x) \cos nx \, dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x) \sin nx \, dx$$

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$$\int_{-\pi}^{+\pi} F(x)\,dx = \frac{a_0}{2}\int_{-\pi}^{\pi} dx + \sum_{n=1}^{\infty}\Big[\underbrace{\cancel{\int_{-\pi}^{\pi} a_n \cos nx\,dx}}_{\to 0}$$

$$+ \underbrace{\cancel{\int_{-\pi}^{\pi} b_n \sin nx\,dx}}_{\to 0}\Big]$$

$$= \frac{a_0}{2}\, x \Big|_{-\pi}^{\pi}$$

$$= \frac{a_0}{2}\left[\pi - (-\pi)\right]$$

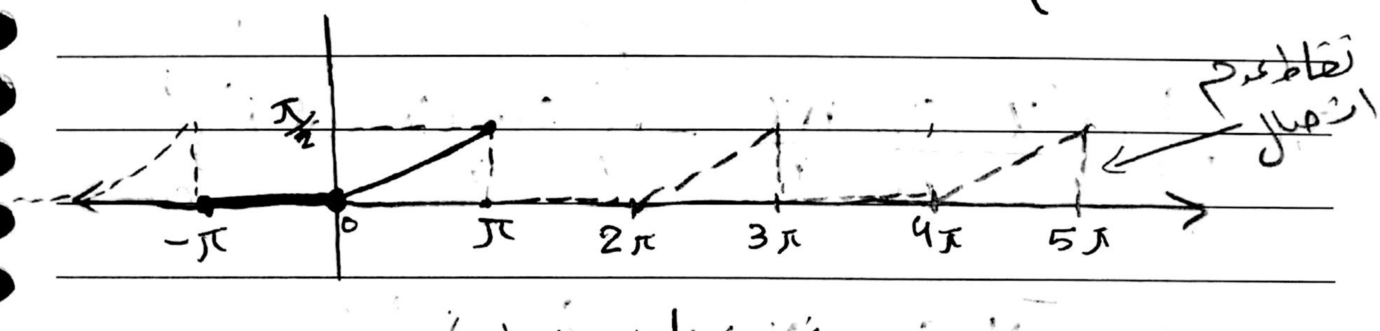
$$\int_{-\pi}^{\pi} F(x)\,dx = \boxed{\pi a_0}$$

Ex: $F(x) = \begin{cases} 0 & -\pi \leq x < 0 \\ \dfrac{x}{2} & 0 < x < \pi \end{cases}$

, $F(x + 2\pi) = F(x)$

period $= 2\pi$ $\longrightarrow$ اشتغل بالقوانين

① نرسم الدالة



غيرمتصلة عند (0)

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Date: Subject:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx$$

$$= \frac{1}{\pi}\left[\underbrace{\cancel{\int_{-\pi}^{0} 0\,dx}}_{\to 0} + \int_{0}^{\pi} \frac{x}{2}\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{x^2}{4}\right]_0^{\pi} = \frac{x^2}{4\pi}\Big|_0^{\pi}$$

$$= \frac{\pi^2}{4\pi} - \frac{0}{4\pi} = \boxed{\frac{\pi}{4}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{0}^{\pi} \frac{x}{2}\cos nx\,dx\right.$$

$$\left. + \underbrace{\cancel{\int_{-\pi}^{0} 0 \cos nx\,dx}}_{\to 0}\right]$$

by parts

$= $ Let $u = x$ $\quad dv = \cos nx\,dx$

$du = dx \quad v = \frac{\sin nx}{n}$

$$a_n = \frac{1}{2\pi}\left[\frac{x\sin nx}{n} - \int \frac{\sin nx}{n}\,dx\right]_0^{\pi}$$

$$= \frac{1}{2\pi n}\left[x\sin nx + \frac{\cos nx}{n}\right|_0^{\pi}$$

Date: Subject:

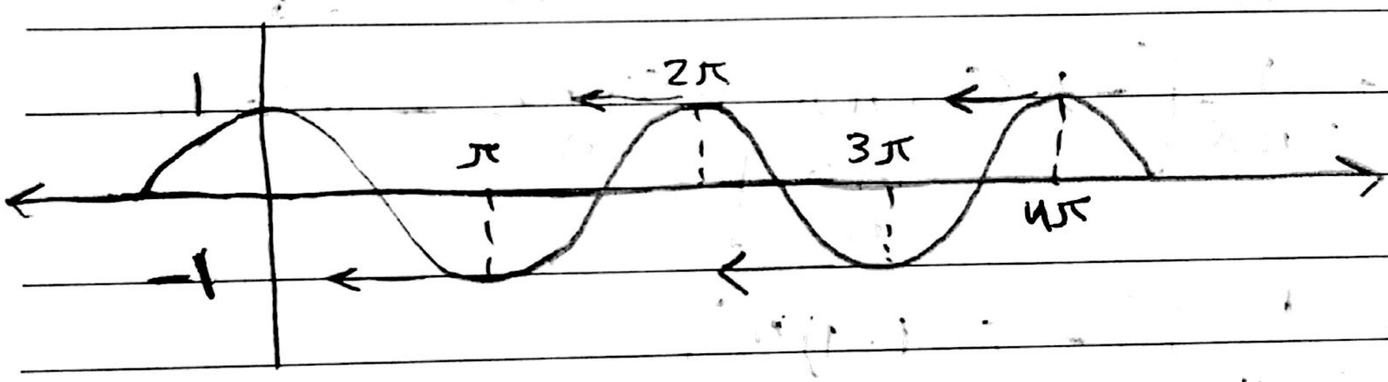
$\sin n\pi = 0$ $\qquad\qquad$ $\cos n\pi = ?$

$\sin 0 = 0$

$$a_n = \frac{1}{2\pi n}\left[\frac{\cos n\pi}{n} - \frac{\cos 0}{n}\right]$$

$$= \frac{1}{2n^2\pi}\left[\cos n\pi - \cos 0\right]$$

$$= \frac{1}{2n^2\pi}\left[(-1)^n - 1\right]$$



لو n فردي $\xrightarrow{\cos} -1$

لو n زوجي $\xrightarrow{\cos} 1$

$\therefore \cos n\pi = (-1)^n$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\sin nx \, dx$$

$$= \frac{1}{\pi}\left[\underbrace{\cancel{\int_{-\pi}^{0} 0 \sin nx \, dx}}_{\to 0} + \int_{0}^{\pi} \frac{x}{2}\sin nx \, dx\right]$$

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Date: Subject:

$$b_n = \frac{1}{2\pi}\left[\int_0^{\pi} x \sin nx \, dx\right]$$

$u = x$ $\quad dv = \sin nx \, dx$

$du = dx$ $\quad v = \frac{-\cos nx}{n}$

x

$-S$

$$b_n = \frac{1}{2\pi}\left[\frac{-x\cos nx}{n} + \int \frac{\cos nx}{n} \, dx\right]_0^{\pi}$$

$$= \frac{1}{2n\pi}\left[-x\cos nx + \int \cos nx \, dx\right]_0^{\pi}$$

$$= \frac{1}{2n\pi}\left[-x\cos nx + \frac{\sin nx}{n}\right]_0^{\pi}$$

$$= \frac{1}{2n\pi}\left[-\pi\cos n\pi + \underbrace{\frac{\sin n\pi}{n}}_{0} - \underbrace{0\cos 0}_{0} - \underbrace{\frac{\sin 0}{n}}_{0}\right]$$

$$= \frac{1}{2\pi n}\left[-\pi(-1)^n\right]$$

$$= \frac{-1}{2n}(-1)^n$$

$$F_x = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$$= \frac{\pi}{8} + \sum_{n=1}^{\infty}\left[\frac{1}{2\pi n^2}\left((-1)^n - 1\right)(\cos nx) + \frac{-1}{2n}(-1)^n(\sin nx)\right]$$